Structural Calculations

Given Material Properties

$Es := 29000000psi$

$Ec := 57000\sqrt{\frac{fc}{(psi)}}\cdot(psi)$

$Eci := 57000\sqrt{\frac{fci}{psi}}\cdot psi$

$n := \frac{Es}{Ec}$ $\quad ni := \frac{Es}{Eci}$

$\varepsilon_y := \frac{f_y}{Es} = 2.069 \times 10^{-3}$

$fci := 5530psi$ $\quad \lambda := 0.75$

$fc := 8330psi$

$f_y := 60000psi$

$fr := 7.5\lambda\sqrt{\frac{fc}{(psi)}}\cdot(psi) = 513.387\ psi$

$dp := \frac{1}{2}in$ $\quad ds := \frac{5}{8}in$

$npb := 2$

$Ap := npb\cdot.153in^2 = 0.306\cdot in^2$

$CCs := 1in$

$CCp := 1in$

$kip \equiv 1000lbf$

$ncs := 2$

$As := ncs.31in^2 = 0.62\cdot in^2$

Variables

Es: Modulus of elasticity of reinforcement
Ec: Modulus of elasticity of concrete
Eci: Modulus of elasticity of concrete at time of initial prestress
fr: Modulus of rupture of concrete
fy: Specified yield strength of reinforcement
fc: Specified compressive strength of concrete
fci: Specified compressive strength of concrete at time of initial prestress

ds: Nominal diameter of bar
dp: Nominal diameter of prestressing steel
CCs: Clear cover of reinforcement
CCp: Clear cover of reinforcement
n: Elastic modulus ratio
ni: Elastic modulus ratio at time of initial prestress
npb: number of prestressed bars
ncs: number of compression steel rebar
ε_y: Strain of reinforcement

Given Geometry

$b1 := 10in$ $\quad h1 := 2.5in$

$b2 := 2.5in$ $\quad h2 := 14in$

$b3 := 10in$ $\quad h3 := 2.5in$

$L \equiv 17ft$

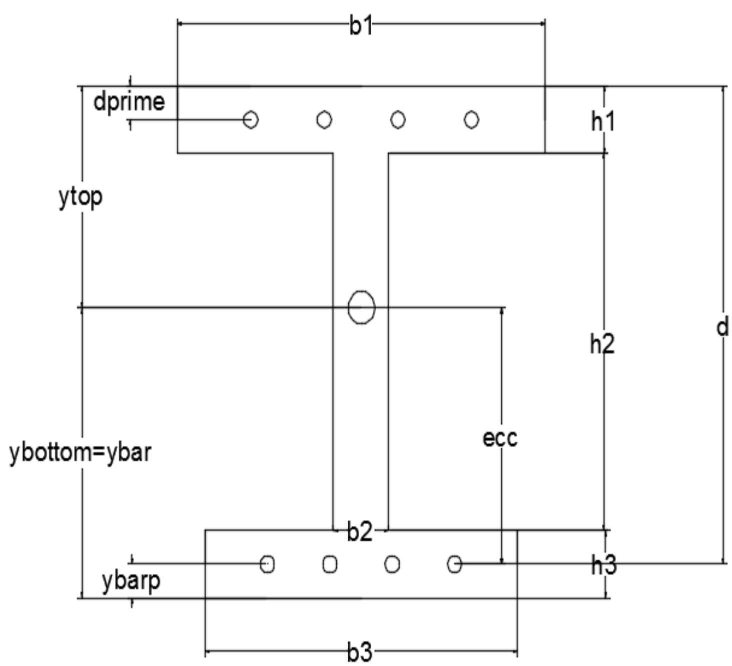
$Lt := 19ft$

$h := h1 + h2 + h3 = 19\cdot in$

Variables

See diagram.

L: unsupported length of beam
Lt: total length of beam



Structural Calculations

Gross Concrete Section Properties

$A1 := b1 \cdot h1 = 25 \cdot in^2$

$A2 := b2 \cdot h2 = 35 \cdot in^2$

$A3 := b3 \cdot h3 = 25 \cdot in^2$

$Ag := A1 + A2 + A3 = 0.59\, ft^2$

$I1 := \frac{b1 \cdot h1^3}{12} = 13.021 \cdot in^4$ $\quad I2 := \frac{b2 \cdot h2^3}{12} = 571.667 \cdot in^4$ $\quad I3 := \frac{b3 \cdot h3^3}{12} = 13.021 \cdot in^4$

$Ig := I1 + I2 + I3 = 597.708 \cdot in^4$

Variables

Ag: Total (gross) cross-sectional area

Beam Selfweight

$\gamma conc := 124.2 \frac{lbf}{ft^3}$ (per mix design)

$weight := \gamma conc \cdot Lt \cdot Ag = 1392.9 \cdot lbf$

$\omega sw := (Ag) \cdot \gamma conc = 73.313 \cdot \frac{lbf}{ft}$

$\omega conc := \gamma conc \cdot Ag = 73.313 \cdot \frac{lbf}{ft}$

$Msw := \frac{\omega sw \cdot L^2}{8} = 2648 \cdot ft \cdot lbf$

$yd := Ag \cdot Lt = 0.415 \cdot yd^3$

Variables

γconc: Concrete unit weight
ωconc: Nominal weight of concrete
Msw: Moment due to the beam's selfweight (ignoring 1'-0" overhang)

Transformed Steel Properties @ Release

$ni - 1 = 5.842$

$At1 := (ni - 1) \cdot Ap = 0.012\, ft^2$

$At2 := (ni - 1) \cdot As = 0.025\, ft^2$

$Atr := At1 + At2 + Ag = 90.409 \cdot in^2$

Variables

Atr: Transformed cross-sectional area

Structural Calculations

Transformed Section Centroid @ Release

$$yg := \frac{[A1\cdot(h2 + h3 + .5h1) + A2\cdot(h3 + .5h2) + A3\cdot(.5h3)]}{A1 + A2 + A3} = 9.5\cdot in$$

$$ybar := \frac{\begin{bmatrix} A1\cdot(h3 + h2 + 0.5h1) + A2\cdot(h3 + 0.5\cdot h2) + A3\cdot(0.5\cdot h3) \ldots \\ + At2\cdot[h3 + h2 + [h1 - (CCs + 0.5ds)]] + At1\cdot(CCp + 0.5\cdot dp) \end{bmatrix}}{(At1 + At2 + A1 + A2 + A3)} = 9.665\cdot in$$

Variables

yg: centroid of gross section
ybar: centroid of transformed section

Transformed Section Moment of Inertia @ Release

$$Itr := \begin{bmatrix} I1 + A1\cdot[(h3 + h2 + 0.5h1) - ybar]^2 \ldots \\ + I2 + A2\cdot(|h3 + 0.5h2 - ybar|)^2 \ldots \\ + I3 + A3\cdot(ybar - 0.5h3)^2 \ldots \\ + At2\cdot[h3 + h2 + [h3 + h3 + [h1 - (CCs + 0.5ds)] - ybar]]^2 \ldots \\ + At1\cdot[ybar - (CCp + 0.5dp)]^2 \end{bmatrix} = 4744\cdot in^4$$

Material Properties @ Release

$fpi := 174ksi$ (assumed)

$Fp := fpi\cdot Ap = 53.2\cdot kip$

$ytop := (h1 + h2 + h3) - ybar = 9.335\cdot in$

$ybottom := ybar = 9.665\cdot in$

$ybarp := .5\cdot h3 = 1.25\cdot in$

$ecc := ybar - ybarp = 8.4\cdot in$

Variables

fpi: Specified conpressive strength of prestress steel at release
Fp: Force in prestressed steel at release
ecc: Eccentricity
ybarp: distance from bottom of flange to prestressed steel

Structural Calculations

Stress at Release

$$\sigma bottom := \frac{-Fp}{Atr} - \frac{(Fp \cdot ecc \cdot ybottom)}{Itr} = -1502\,psi$$

$$\sigma top := \frac{-Fp}{Atr} + \frac{(Fp \cdot ecc \cdot ytop)}{Itr} = 293\,psi$$

(ignore sw at transfer length, conservative)

Check:

$$\left|\frac{\sigma bottom}{.6fci}\right| \le 1 = 1$$ OK

$$\frac{\sigma top}{\sqrt{\frac{fci}{psi}} \cdot psi} = 3.937$$ < 6 OK, provide top steel anyway

$.6fci = 3318\,psi$ OK

Transformed Steel Properties @ 28-days

$n - 1 = 4.574$

$At1 := (n - 1) \cdot Ap = 9.721 \times 10^{-3}\,ft^2$

$At2 := (n - 1) \cdot As = 0.02\,ft^2$

$Atr := At1 + At2 + Ag = 89.236 \cdot in^2$

Transformed Section Centroid @ 28-days

$$ybar := \frac{\begin{bmatrix} A1 \cdot (h3 + h2 + 0.5h1) + A2 \cdot (h3 + 0.5 \cdot h2) + A3 \cdot (0.5 \cdot h3) \ldots \\ + At2 \cdot [h3 + h2 + [h1 - (CCs + 0.5ds)]] + At1 \cdot (CCp + 0.5 \cdot dp) \end{bmatrix}}{(At1 + At2 + A1 + A2 + A3)} = 9.631 \cdot in$$

$$yg := \frac{[A1 \cdot (h2 + h3 + .5h1) + A2 \cdot (h3 + .5h2) + A3 \cdot (.5h3)]}{A1 + A2 + A3} = 9.5 \cdot in$$

Moment of Inertia of Areas 1,2,3

$I1 := \frac{b1 \cdot h1^3}{12} = 13.021 \cdot in^4$ $I2 := \frac{b2 \cdot h2^3}{12} = 571.667 \cdot in^4$ $I3 := \frac{b3 \cdot h3^3}{12} = 13.021 \cdot in^4$

$Ig := I1 + I2 + I3 = 598 \cdot in^4$

Structural Calculations

Transformed Section Moment of Inertia @ 28-days

$$Itr := \begin{bmatrix} I1 + A1\cdot[(h3 + h2 + 0.5h1) - ybar]^2 \ldots \\ + I2 + A2\cdot(|h3 + 0.5h2 - ybar|)^2 \ldots \\ + I3 + A3\cdot(ybar - 0.5h3)^2 \ldots \\ + At2\cdot[h3 + h2 + [h1 - (CCs + 0.5ds)] - ybar]^2 \ldots \\ + At1\cdot[ybar - (CCp + 0.5dp)]^2 \end{bmatrix} = 4285\cdot in^4$$

Cracking Load

$fps := 180000psi$ (assumed)

$Fps := fps\cdot Ap = 55.1\cdot kip$

$$Pcr := \frac{2Itr}{7ft\cdot ybottom}\cdot\left(fr + \frac{Fps}{Atr} + \frac{Fps\cdot ecc\cdot ybottom}{Itr} - \frac{Msw\cdot ybottom}{Itr}\right) = 22.2\cdot kip$$

Check:

$Pcr \geq 20kip = 1$ OK

Variables

fps: Stress in prestressing steel at initial cracking load
Fps: Force in prestressed steel at cracking
Pcr: Cracking Load

Cracking Moment

$$Mcr := \left[fr + \frac{Fps}{Atr} + \frac{(Fps\cdot ecc\cdot ybar)}{Itr} - \left(Msw\cdot\frac{ybar}{Itr}\right)\right]\cdot\frac{Itr}{ybar} = 78\cdot kip\cdot ft$$

Structural Calculations

Ultimate Moment Properties

$\varepsilon c := -0.003 \qquad Ep := 29 \cdot 10^{6} psi \qquad fy := 60000 psi$

$\varepsilon y := \frac{f_y}{Es} = 2.069 \times 10^{-3}$

$\beta 1 := \max\left[\min\left[.85 - (fc - 4000psi)\left(\frac{.05}{1000psi}\right), .85\right], .65\right] = 0.65$

$d := h1 + h2 + \frac{1}{2} h3 = 17.75 \cdot in$

$dprime := CCs \qquad cz := 1in$

Given

$$\left[-.85fc \cdot \beta 1 \cdot cz \cdot b1 + \max\left[\varepsilon c \cdot \frac{(cz - dprime)}{cz} \cdot Es, -fy\right] \cdot As \ldots + Ap \cdot \min\left[Ep \cdot \left[0.003 \cdot \frac{(d - cz)}{cz}\right] \cdot \left[0.025 + \frac{0.975}{\left[1 + \left[118 \cdot \left[0.003 \cdot \frac{(d - cz)}{cz}\right]\right]^{10}\right]^{\left(\frac{1}{10}\right)}}\right], 270ksi \right] \right] = 0$$

$cz := \text{Minerr}(cz) = 1.416 \cdot in$

$\varepsilon sprime := \varepsilon c \cdot \frac{(cz - dprime)}{cz} = -8.811 \times 10^{-4}$

$$fsprime := \begin{cases} (\varepsilon sprime \cdot Es) & \text{if } |\varepsilon sprime| \le \varepsilon y \\ fy & \text{otherwise} \end{cases} = -25.551 \cdot ksi$$

$\varepsilon p := 0.003 \cdot \frac{(d - cz)}{cz} = 0.035$

$$fpu := \min\left[Ep \cdot \varepsilon p \cdot \left[0.025 + \frac{0.975}{\left[1 + (118 \cdot \varepsilon p)^{10}\right]^{\left(\frac{1}{10}\right)}}\right], 270ksi \right] = 265 \cdot ksi$$

$T := Ap \cdot fpu = 81001.8\, lbf$

$Cs := fsprime \cdot As = -15841.6\, lbf \qquad Cc := -.85fc \cdot \beta 1 \cdot cz \cdot b1 = -65160.1\, lbf$

$Fpu := T$

Check:

$Cs + Cc + T = -0\, lbf$ OK

Structural Calculations

Variables

εc: Strain of concrete
εp: Strain of Prestressing steel
Ep: Modulus of elasticity of prestressing steel
$\beta 1$: factor relating depth of equivalent rectangular compressive stress block to neutral axis depth
fpu: Specified tensile strength of prestressing steel
Cs: Compressionforce from steel
Cc: Compression force from concrete
Fpu: Force in the prestressed steel at ultimate load
cz: depth of compression zone

Ultimate Moment

$$a := \beta 1 \cdot cz = 0.92 \cdot in$$

$$jd := d - \frac{a}{2} = 17.29 \cdot in$$

$$Mn := -Cc \cdot \left(cz - \frac{a}{2}\right) - Cs \cdot (cz - dprime) + T \cdot (d - cz) = 116 \cdot kip \cdot ft$$

Ultimate Load

$$Pu := (Mn - Msw) \cdot \frac{2}{7ft} = 32.4 \cdot kip$$

Check:

$40kip \ge Pu \ge 32kip = 1$ OK

Approximated Ultimate Deflection

$$\delta := \frac{Pu \cdot (L)^3}{48Ec \cdot Ig} = 1.842 \cdot in$$

$$\theta := \sigma top \cdot Ag = 24.882 \cdot kip$$

NOTE: See deflection calculations for accurate deflection

Check:

$As > \frac{\theta}{40ksi} = 0$ OK

Variables

δ: Ultimate deflection
θ: Rotation
As: Area of nonprestressed longitudinal tension reinforcement

Shear Calculations

Recall: $\gamma conc = 124.2 \cdot \frac{lbf}{ft^3}$ $\omega conc = 73.313 \cdot \frac{lbf}{ft}$ $Ag = 0.59\ ft^2$ $\sqrt{\frac{fc}{(psi)}} \cdot (psi)$

$h = 1.583\ ft$

$W2 := \omega conc \cdot L = 1246\ lbf$

$By := \frac{Pu}{2} + \frac{W2}{2} = 16.8 \cdot kip$ $Ay := By = 16.816 \cdot kip$

Variables:

W2: Total load due to selfweight
Ay: Reaction at A
By: Reaction at B

Shear at h/2 from support

$v1 := Ay - \omega conc \cdot \frac{h}{2} = 16.758 \cdot kip$ $M1 := Ay \cdot \frac{h}{2} - \omega conc \cdot \frac{h^2}{8} = 13.3 \cdot kip \cdot ft$

Shear @ 7'-0" From Support

$v2 := Ay - \omega conc \cdot 7ft = 16.3 \cdot kip$ $M2 := 7ft \cdot Ay + \omega conc \cdot 24.5ft^2 = 120 \cdot kip \cdot ft$

Shear Capacity of Reinforcement

$Av := 0.080in^2$ $fyt := 60000psi$ $sp := 4in$

$Vs := \frac{(Av \cdot fyt \cdot d)}{sp} = 21.3 \cdot kip$

Variables:

Av: Area of shear reinforcement with spacing "sp"
fyt: Specified yield strength "fy" of transverse reinforcement
Vs: Nominal shear strength provided by shear reinforcement

Structural Calculations

Shear and Moment at h/2 due to dead load only

$$Mdeadsupport := \omega conc \cdot \frac{L}{2} \cdot \left(\frac{h}{2}\right) - \omega conc \cdot \frac{\left(\frac{h}{2}\right)^2}{2} = 0.47 \cdot kip \cdot ft$$

$$Vd := \omega conc \cdot \frac{L}{2} - \omega conc \cdot \frac{h}{2} = 565\ lbf$$

$$\theta := \sigma top \cdot Ag = 24.9 \cdot kip$$

Check:

$$As > \frac{\theta}{40ksi} = 0$$ OK

Shear Capacity of Concrete at h/2

$$fpe := -\sigma bottom = 1501.7\ psi$$

$$fd := Mdeadsupport \cdot \frac{yg}{Ig} = 89.7\ psi$$

$$Vi := v1 = 16.758 \cdot kip$$

$$Mn = 116 \cdot kip \cdot ft$$

$$Mmax := M1 = 13.3 \cdot kip \cdot ft$$

$$Mcre := \left(\frac{Ig}{yg}\right) \cdot \left[6 \cdot \lambda \cdot \left[\sqrt{\frac{fc}{(psi)}} \cdot (psi)\right] + fpe - fd\right] = 9.6 \cdot kip \cdot ft$$

$$bw := b2 = 2.5 \cdot in \qquad d = 1.479\ ft$$

$$Vci := 0.6 \cdot \lambda \cdot \left[\sqrt{\frac{fc}{(psi)}} \cdot (psi)\right] \cdot d \cdot bw + Vd + \frac{(Vi \cdot Mcre)}{Mmax} = 14.4 \cdot kip$$

$$Vcw := \left[3.5 \cdot \lambda \cdot \left[\sqrt{\frac{fc}{(psi)}} \cdot (psi)\right]\right] \cdot bw \cdot d = 10.6 \cdot kip$$

$$Vc := \min(Vci, Vcw) = 10.6 \cdot kip$$

$$Vn := Vc + Vs = 31.9 \cdot kip$$

Check:

$$\frac{Vs}{\lambda \cdot \left[\sqrt{\frac{fc}{(psi)}} \cdot (psi)\right] \cdot bw \cdot d} = 7.012$$ < 8 OK

Structural Calculations

Shear Capacity of Concrete at h/2 (cont)

Check:

$Vn > v1 = 1$ OK

Variables:

Mcre: Moment causing flexural cracking at section due to externally applied loads
Vci: Nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment
Vcw: Nominal shear strength provided by concrete when diagonal cracking results from high principal tensile stress in web
Vc: Nominal shear strength provided by concrete
Vn: Nominal shear strength
Vu: factored shear force at section

Shear Capacity of Concrete @ 7'-0" from support

$$Mdeadload := \omega conc\cdot\frac{L}{2}\cdot(7ft) - \omega conc\cdot\frac{(7ft)^2}{2} = 2.566\cdot kip\cdot ft$$

$$Vd := \omega conc\cdot\frac{L}{2} - \omega conc\cdot 7ft = 109.969\ lbf$$

$$fpe := -\sigma bottom = 1501.7\ psi \qquad fd := Mdeadload\cdot\frac{yg}{Ig} = 489\ psi$$

$$Vi := v2 = 16.3\cdot kip \qquad Mn = 116\cdot kip\cdot ft$$

$$Mcre := \left(\frac{Ig}{yg}\right)\cdot\left[6\cdot\lambda\cdot\left[\sqrt{\frac{fc}{(psi)}}\cdot(psi)\right] + fpe - fd\right] = 7.5\cdot kip\cdot ft$$

$$bw := b2 = 2.5\cdot in \qquad d = 1.479\ ft$$

$$Vci := 0.6\cdot\lambda\cdot\left[\sqrt{\frac{fc}{(psi)}}\cdot(psi)\right]\cdot d\cdot bw + Vd + \frac{(Vi\cdot Mcre)}{Mmax} = 2.95\cdot kip \qquad Mmax := M2 = 1.195\times 10^{5}\cdot lbf\cdot ft$$

$$Vcw := \left[3.5\cdot\lambda\cdot\left[\sqrt{\frac{fc}{(psi)}}\cdot(psi)\right]\right]\cdot bw\cdot d = 10.6\cdot kip$$

$$Vc := \min(Vci, Vcw) = 3\cdot kip$$

$$Vn := Vc + Vs = 24.3\cdot kip$$

Structural Calculations

Shear Capacity of Concrete @ 7'-0" (cont)

Check:

$$\frac{Vs}{\lambda \cdot \left[\sqrt{\frac{fc}{(psi)}} \cdot (psi)\right] \cdot bw \cdot d} = 7.012$$ < 8 OK

$Vn > v2 = 1$ OK

Check Development Length

$$ld := \left(\frac{fpi}{3000psi}\right) \cdot dp + \frac{(fpu - fpi)}{1000psi} \cdot dp = 6.196\ ft$$ < 7 ft OK

Structural Calculations

Cost Calculations

$dollar \equiv 1¤$

$Amesh := 0.12 \frac{in^2}{ft}$

$yd \equiv 3ft$

$costpervolume := 110 \frac{dollar}{yd^3}$

$totalareaformwork := (h + b3) \cdot Lt = 45.917\, ft^2$

$totalvolume := Lt \cdot (Ag) = 0.415 \cdot yd^3$

$costperlengthprestressed := .30 \frac{dollar}{ft}$

$costperpoundnumber5 := .45 \frac{dollar}{lbf}$

$costperpoundmesh := .50 \frac{dollar}{}$

$costpersqfootformwork := 1.25 \frac{dollar}{ft^2}$

$totalcostconcrete := costpervolume \cdot totalvolume = 45.69 \cdot dollar$

$totalcostprestressed := Lt \cdot costperlengthprestressed \cdot npb = 11.4 \cdot dollar$

$totalcostnumber5 := 1.04 \frac{lbf}{ft} \cdot ncs\, Lt \cdot (costperpoundnumber5) = 17.784 \cdot dollar$

$totalcostmesh := 2 \cdot Amesh \cdot Lt \cdot 0.85 \frac{lbf}{ft^2} \cdot (costperpoundmesh) = 0.013 \cdot dollar$

$totalcostformwork := [(h + b3) \cdot Lt] \cdot costpersqfootformwork = 57.396 \cdot dollar$

$totalcost := totalcostprestressed + totalcostconcrete\ ...\ + totalcostnumber5 + totalcostmesh + totalcostformwork = 132.285 \cdot dollar$

Weight

$concreteweight := \gamma conc \cdot totalvolume = 1.393 \times 10^3\, lbf$

$steelweight := ncs\, Lt \cdot 1.04 \frac{lbf}{ft} + npb\, Lt \cdot .52 \frac{lbf}{ft} = 59.28 \cdot lbf$

$meshweight := 2 \cdot Amesh \cdot Lt \cdot 0.85 \frac{lbf}{ft^2} = 0.027 \cdot lbf$

$totalweight := steelweight + concreteweight + meshweight = 1452 \cdot lbf$

Structural Calculations

<u>Structural Calculations</u>